# ABSTRACT


1. **Practical Problem:** Test whether it is MORE PROBABLE THAN NOT that the defendant's action was a NECESSARY CAUSE for the plaintiff's injury (or death).

2. **Theoretical Problems:**
   (a) What is the meaning of $PN(x, y)$:
   "Probability that event $y$ would not have occured if it were not for event $x$, given that $x$ and $y$ did in fact occur."
   (b) Under what conditions can $PN(x, y)$ be estimated from statistical data, i.e., observational, experimental and combined.

## REVIEW OF COUNTERFACTUALS

1. **Semantics:** $Y_x(u) \triangleq$ solution for $Y$ in $M_x$

   $Y_x(u) = y$ : $Y = y$ if $X$ were $x$ (in background $u$)

2. **Abreviations:** $Y_x(u) = y \Leftrightarrow y_x(u)$ or $y_x$

$$P(Y_x = y) \Leftrightarrow P(y_x)$$

$$y' \triangleq \text{ complement of } y$$

e.g.,

$$\begin{aligned} y'_x &\equiv Y_x \neq y \\ y_x \vee y'_x &\equiv true \\ P(y_x|x) &= P(y|x) \\ P(y'_x) &= 1 - P(y_x) \end{aligned}$$

# THE SEMANTICS OF NECESSARY AND SUFFICIENT CAUSES

**Necessary Cause**
Event $x$ was a necessary cause of event $y$ if the probability

$$PN = P(y'_{x'}|x, y) \text{ is HIGH}$$

**Sufficient Cause**
Event $x$ is a sufficient cause of event $y$ if the probability

$$PS = P(y_x|x', y') \text{ is HIGH}$$

(e.g., benefit of treating the untreated sick)

**Necessary and Sufficient Cause** Event $x$ is a necessary-and-sufficient cause of event $y$ if the probability

$$PNS = P(y_x, y'_{x'}) \text{ is HIGH}$$

# EXOGENEITY

**Definition 1** (no confounding = **exogeneity**)

$M =$ *model of the data-generating process.*

$P_M(y|do(x)) =$ *probability of* $Y = y$ *under the hypothetical intervention* $X = x$*.*

*We say that* $X$ *and* $Y$ *are* **not confounded** *in* $M$ *(or,* $X$ *is* **exogenous***) if and only if*

$$P_M(y|do(x)) = P_M(y|x)$$

Alternatively,

$$P_M(y_x) \triangleq P_M(Y_x = y) = P_M(y|x)$$

$Y_x-$ the value of $Y$ if $X$ were $x$

[Neyman, 1926; Rubin, 1974]

## BOUNDS AND BASIC RELATIONSHIPS

What if we do not have the functional relationships behind the data-generation mechanisms?

What can be done with joint distribution alone?

**Theorem 1:**
Under condition of exogeneity, PNS is bounded as follows:

$$\max\ [0, P(y|x) + P(y'|x') - 1] \leq PNS$$
$$PNS \leq \min[P(y|x), P(y'|x')]$$

and

$$PN = \frac{PNS}{P(y|x)} \qquad PS = \frac{PNS}{1 - P(y|x')}$$

**Interpretation:**
The probability of necessity can take on any value in the interval

$$ERR \triangleq 1 - \frac{1}{RR} \triangleq \frac{P(y|x) - P(y|x')}{P(y|x)} \leq PN \leq 1$$

# IDENTIFIABILITY UNDER EXOGENEITY AND MONOTONICITY

**Theorem 3**
If $X$ is exogenous and $Y$ is monotonic in $X$, then the probabilities PN, PS, and PNS are all identifiable, and are given by:

$$PNS = P(y|x) - P(y|x') \quad \text{risk-difference}$$

$$PN = [P(y|x) - P(y|x')]/P(y|x) \quad \text{excess-risk-ratio}$$

$$PS = [P(y|x) - P(y|x')]/P(y'|x') \quad \text{susceptibility}$$

**Interpretation:**
There is wisdom to epidemiological myths, BUT:

1. Caution: We need to ascertain monotonicity (no prevention, no reversal)
2. Relief: No need to assume independence (between susceptibility and background factors)

# BOUNDS UNDER EXOGENEITY AND NONMONOTONICITY

**Theorem** $3'$ (Tian & Pearl 2000)
If $X$ is exogenous then the probabilities PN, PS, and PNS are all
**Lower Bounded by:**

$$
\begin{array}{rcll}
PNS & \geq & P(y|x) - P(y|x') & \text{risk-difference} \\
PN & \geq & [P(y|x) - P(y|x')]/P(y|x) & \text{excess-risk-ratio} \\
PS & \geq & [P(y|x) - P(y|x')]/P(y'|x') & \text{susceptibility}
\end{array}
$$

**Interpretation:**
There is wisdom to epidemiological myths, since we need **not** ascertain monotonicity (no prevention, no reversal) **for lower bounding PN**.

# WHEN IS THE PROBABILITY OF CAUSATION IDENTIFIABLE?

**Theorem:** If $Y$ is **monotonic** in $X$, then the probabilities of causation PNS, PN and PS are identifiable whenever the **effect of action** $P(y_x)$ is identifiable, and are given by:

$$PNS = P(y_x) - P(y_{x'})$$

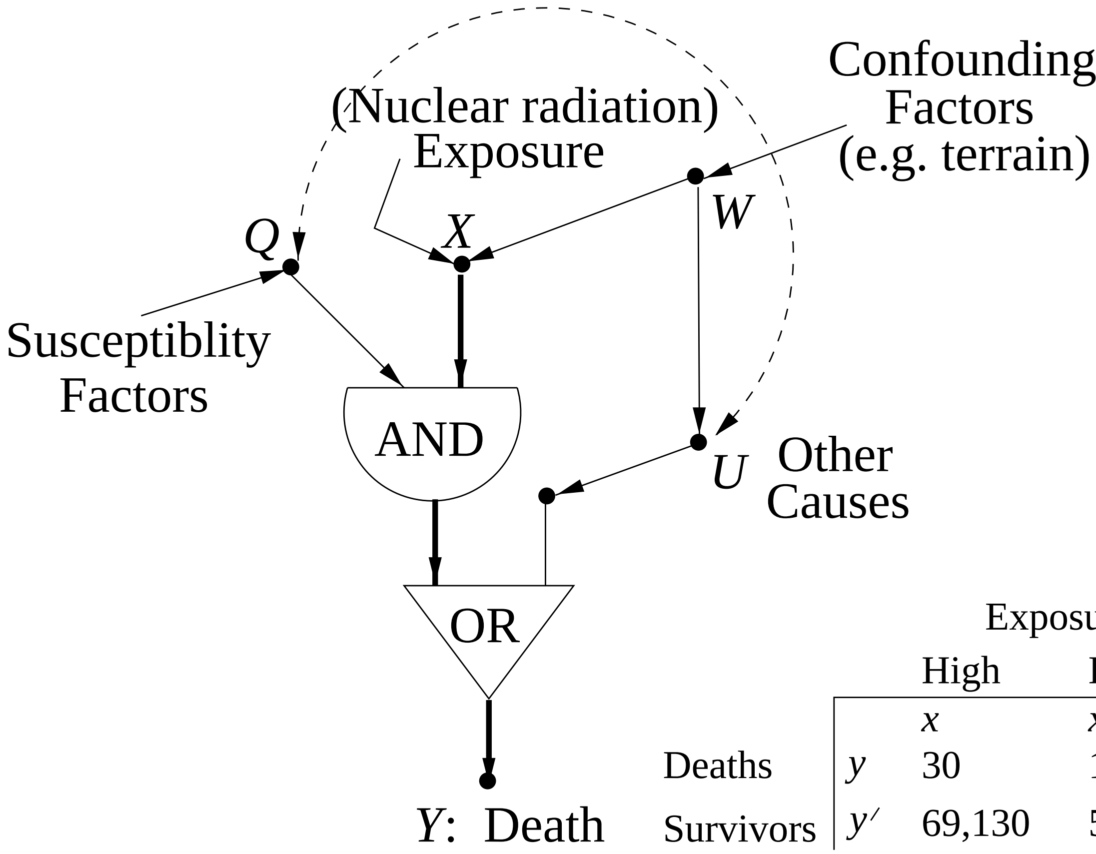
$$PN = \frac{P(y|x) - P(y|x')}{P(y|x)} + \frac{P(y|x') - P(y_{x'})}{P(x,y)}$$

$$PS = \frac{P(y|x) - P(y|x')}{P(y'|x')} + \frac{P(y_x) - P(y|x)}{P(x',y')}$$

**Note:** $P(y_x) = P(Y = y|do(X = x))$ is identifiable

1. in experimental studies,
2. when $x$ and $y$ are not confounded, or
3. when $x$ and $y$ are unconfounded through adjustment for covariates (given $G(M)$).

# EXAMPLE: WHEN IS A DISEASE ATTRIBUTABLE TO EXPOSURE?




| | | Exposure | |
|---|---|---|---|
| | | High | Low |
| | | $x$ | $x'$ |
| Deaths | $y$ | 30 | 16 |
| Survivors | $y'$ | 69,130 | 59,010 |

**Q.** What is the probability PN that a child who died from leukemia after exposure would have survived had he/she not been exposed ?

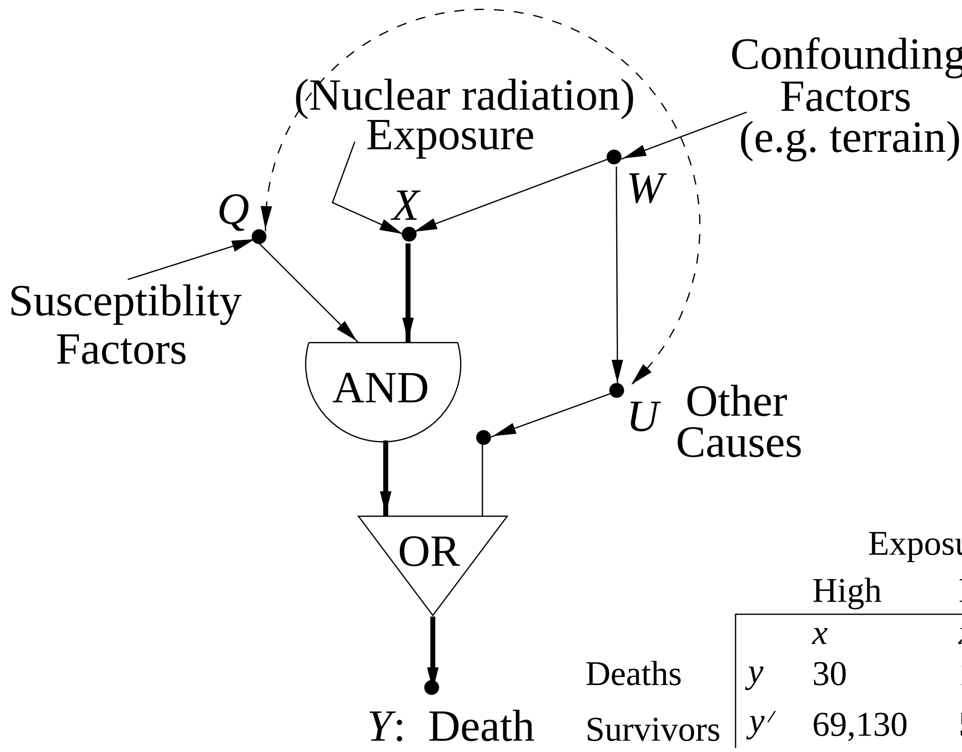
**A.**

$$PN = ERR \triangleq [P(y|x) - P(y|x')]/P(y|x)$$

$$PN = ERR + [P(y|x') - P(y_{x'})]/P(x,y)$$

$$P(y_{x'}) = \sum_w P(y|w,x')P(w)$$

# WHEN IS A DISEASE ATTRIBUTABLE TO EXPOSURE? (Cont.)




| | | Exposure | |
|---|---|---|---|
| | | High | Low |
| | | $x$ | $x'$ |
| Deaths | $y$ | 30 | 16 |
| Survivors | $y'$ | 69,130 | 59,010 |

**Numerical computation** (assuming no confounding):

$$PNS = P(y|x) - P(y|x') = \frac{30}{30+69,130} - \frac{16}{16+59,010} = .0001625 \quad \textbf{(9.45)}$$

$$PN = \frac{PNS}{P(y|x)} = \frac{PNS}{30/(30+69,130)} = .37535 \quad \textbf{(9.46)}$$

$$PS = \frac{PNS}{1-P(y|x')} = \frac{PNS}{1-16/(16+59,010)} = .0001625 \quad \textbf{(9.47)}$$

# LEGAL RESPONSIBILITY FROM EXPERIMENTAL AND NON-EXPERIMENTAL DATA

- A lawsuit is filed against the manufacturer of drug $x$, charging that the drug is likely to have caused the death of Mr. A, who took the drug to relieve symptom $S$ associated with disease $D$.

- An experimental study shows only minor increase in death rates among drug users.

- The plaintiff argues, however, that the experimental study is of little relevance to this case, because it represents the effect of the drug on *all* patients, not on patients like Mr. A who actually died while using drug $x$.

- Moreover, argues the plaintiff, Mr. A is unique in that he used the drug on his own volition, unlike subjects in the experimental study who took the drug to comply with experimental protocols.

# LEGAL RESPONSIBILITY FROM EXPERIMENTAL AND NON-EXPERIMENTAL DATA

Find *PN*=*P*(*drug* $x$ *is the cause of Mr.* $A$*'s death*)

Experimental

| | | $x$ | $x'$ |
|---|---|---|---|
| Deaths | $y$ | 16 | 14 |
| Survivals | $y'$ | 984 | 986 |

Non-Experimental

| | | $x$ | $x'$ |
|---|---|---|---|
| Deaths | $y$ | 2 | 28 |
| Survivals | $y'$ | 998 | 972 |

Defendant : $\dfrac{P(y_x) - P(y_{x'})}{P(y_x)} = \dfrac{0.016 - 0.014}{0.016} = 0.125$

**Plaintiff:** Mr. $A$ is not a typical subject,
he **chose** the drug ($x$), and **died** ($y$).

Defendant: non-experimental data is biased.

**Plaintiff:** $PN \geq \dfrac{P(y) - P(y_{x'})}{P(y, x)} = \dfrac{0.015 - 0.014}{0.001} = 1$

Jury: Guilty! Combined data tell more than each study alone (monotonicity not assumed).

# TESTABLE IMPLICATIONS OF NO-PREVENTION

- If $x$ cannot prevent $y$, then every combination of experimental and nonexperimental data, taken from the same population, must satisfy the inequalities:

  $$P(x', y) \leq P(y_{x'}) \leq P(y) \leq P(y_x) \leq 1 - P(x, y')$$

- If the inequalities are violated, then the data are not drawn from the same population.

# SUMMARY OF RESULTS

1. Formal semantics for $PN(x, y)$

2. Exogeneity and monotonicity are needed for
$$ERR = [P(y|x) - P(y|x')]/P(y|x)$$
to be an unbiased estimator of $PN(x, y)$

3. Bounds under exogeneity and NON-monotonicity

4. Under NON-exogeneity and monotonicity, experimental data alone are useless. Combined experimental and observational data permit unbiased estimation of $PN(x, y)$

5. Correction for confounding yield unbiased estimation of $PN$ under monotonicity, and bounds on $PN$ without monotonicity.

Reference: Tech Report R-271 (Tian & Pearl 2000)
http://www.cs.ucla.edu/~judea

## $PN$ AS A FUNCTION OF ASSUMPTIONS AND AVAILABLE DATA

| **Assumptions** | | **Data Available** | | |
|---|---|---|---|---|
| Exo. | Mono. | Exp. | Non-exp. | Combined |
| + | + | ERR | ERR | ERR |
| + | − | bounds | bounds | bounds |
| − | + | — | — | CERR |
| − | − | — | — | bounds |

Note: CERR stands for corrected ERR.

## BOUNDING BY LP
## (After Tian & Pearl 2000, R-271)

**Parameters:**

$$
\begin{aligned}
p_{111} = &\ P(y_x, y_{x'}, x) &= P(x, y, y_{x'}) \\
p_{110} = &\ P(y_x, y_{x'}, x') &= P(x', y, y_x) \\
p_{101} = &\ P(y_x, y'_{x'}, x) &= P(x, y, y'_{x'}) \\
p_{100} = &\ P(y_x, y'_{x'}, x') &= P(x', y', y_x) \\
p_{011} = &\ P(y'_x, y_{x'}, x) &= P(x, y', y_{x'}) \\
p_{010} = &\ P(y'_x, y_{x'}, x') &= P(x', y, y'_x) \\
p_{001} = &\ P(y'_x, y'_{x'}, x) &= P(x, y', y'_{x'}) \\
p_{000} = &\ P(y'_x, y'_{x'}, x') &= P(x', y', y'_x)
\end{aligned}
$$

**Maximize (Minimize):**

$$PNS = p_{101} + p_{100} \quad \textbf{(9.18)}$$

$$PN = p_{101} / P(x, y) \quad \textbf{(9.19)}$$

$$PS = p_{100} / P(x', y') \quad \textbf{(9.20)}$$

# BOUNDING BY LP
# (Cont.)

**Probabilistic constraints:**

$$\sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1} p_{ijk} = 1$$

$$p_{ijk} \geq 0 \text{ for } i, j, k \in \{0, 1\} \qquad \textbf{(9.15)}$$

**Nonexperimental constraints:**

$$\begin{aligned} p_{111} + p_{101} &= P(x, y) \\ p_{011} + p_{001} &= P(x, y') \\ p_{110} + p_{010} &= P(x', y) \end{aligned} \qquad \textbf{(9.16)}$$

**Experimental constraints:**

$$\begin{aligned} P(y_x) &= p_{111} + p_{110} + p_{101} + p_{100} \\ P(y_{x'}) &= p_{111} + p_{110} + p_{011} + p_{010} \end{aligned} \qquad \textbf{(9.17)}$$

# FROM COUNTERFACTUALS TO PERSONAL DECISION MAKING

| | Experimental | | Nonexperimental | |
|---|---|---|---|---|
| | $x$ | $x'$ | $x$ | $x'$ |
| Deaths ($y$) | 16 | 14 | 2 | 28 |
| Survivals ($y'$) | 984 | 986 | 998 | 972 |
| | 1,000 | 1,000 | 1,000 | 1,000 |

- Mr. $B$, survived without drug.
  Would he risk death by starting now?

**Nonexperimental data:** $P(y|x) = 0.002$

**Experimental data:** $P(y_x) = 0.016$

**Answer:** Risk $= PS = P(y_x|x', y')$

**Bounded by:**

$$\frac{P(y_x) - P(y)}{P(x', y')} \leq PS \leq \frac{P(y_x) - P(x, y)}{P(x', y')}$$

$$0.002 \leq PS \leq 0.031$$

**Assuming monotonicity (no curing):** $PS = 0.002$

# FROM COUNTERFACTUALS TO TEMPORAL REASONING

When is

$P($future outcome$|$current action, past conditions and actions$)$

reducible to

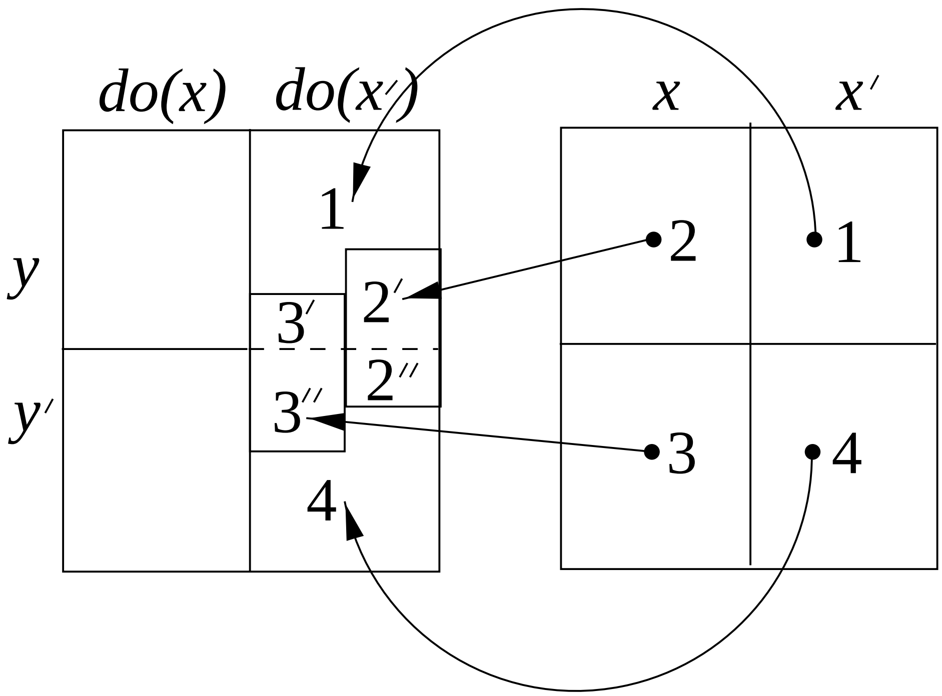
$P($present outcome $|$ hypothetical past action,
actual past conditions and actions)

Symbolically, when can we assume the equalities

$$
\begin{aligned}
&P(y(t+1)|do(x(t)), x'(t-1), y(t)) \\
&\quad = P(y_{x(t)}(t+e)|x'(t-e), y'(t)) \\
&\quad = P(y_x|x', y')
\end{aligned}
$$

# HOW DATA CAN UNCOVER THE TRUE CAUSE OF DEATH

| | Experimental | | Nonexperimental | |
|---|---|---|---|---|
| | $do(x)$ | $do(x')$ | $x$ | $x'$ |
| Deaths ($y$) | 16 | 14 | 2 | 28 |
| Survivals ($y'$) | 984 | 986 | 998 | 972 |
| | 1,000 | 1,000 | 1,000 | 1,000 |




$$P(y_{x'}) = \frac{n_1 + n_{2'} + n_{3'}}{n_1 + n_2 + n_3 + n_4} = \frac{14}{1000}$$

$$P(x', y) = \frac{n_1}{n_1 + n_2 + n_3 + n_4} = \frac{28}{2000}$$

$$n'_2 + n'_3 = 0 \Rightarrow P(y_{x'}, x) = 0 \quad \Rightarrow P(y'_{x'}|x) = 1$$