BOUNDING TREATMENT EFFECTS (EXAMPLE)

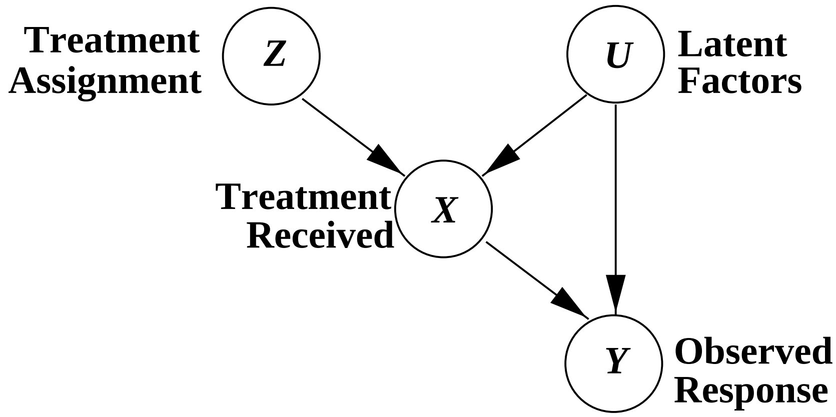


Figure 8.1:

1. Pre-intervention distribution

$$P(y,x,z,u) = P(y|x,u)P(x|z,u)P(z)P(u)$$

2. Post-intervention distribution, $x \to \hat{x}$

$$P(y,z,u|\hat{x}) = P(y|x,u)P(z)P(u)$$

3. Bound treatment effect (from 2)

$$P(y|\hat{x}) = \sum_u P(y|x,u)P(u)$$

Subject to given observed distribution (from 1)

$$P(y,z,x) = \sum_u P(y|x,u)P(x|z,u)P(z)P(u)$$

using $|dom(U)| = 16$

REDUCING THE DOMAIN OF U

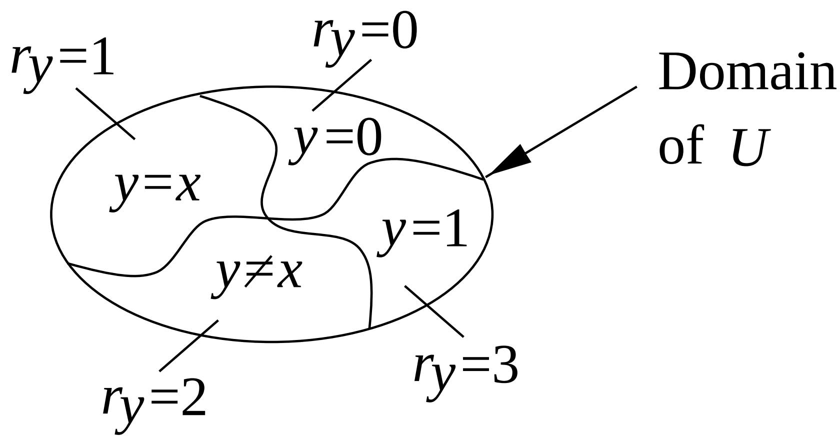


Figure 8.2: The partition of U into four equivalence classes, each inducing a distinct functional mapping from X to Y for any given function $y = f(x, u)$.

Consider the structural equation that connects two binary variables, Y and X, in a causal model:

$$y = f(x, u).$$

For any given u, the relationship between X and Y must be one of four functions:

$$\begin{aligned} f_0 &: y = 0, & f_1 &: y = x, \\ f_2 &: y \neq x, & f_3 &: y = 1. \end{aligned} \qquad \textbf{(8.5)}$$

$P(u)$ translates into $P(r), r = 0, 1, 2, 3$.

MINIMAL-STATE STRUCTURE

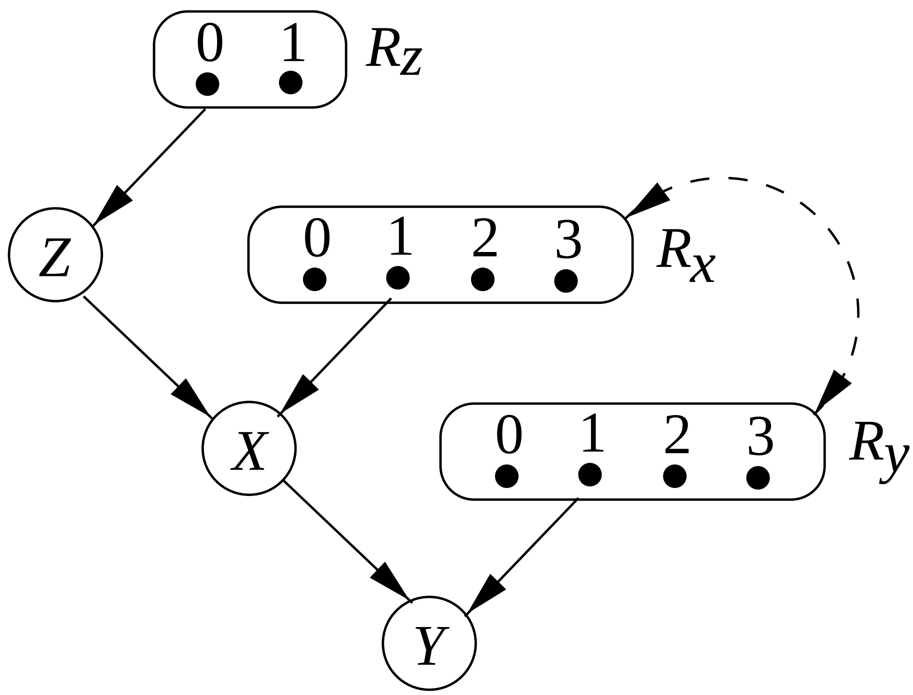
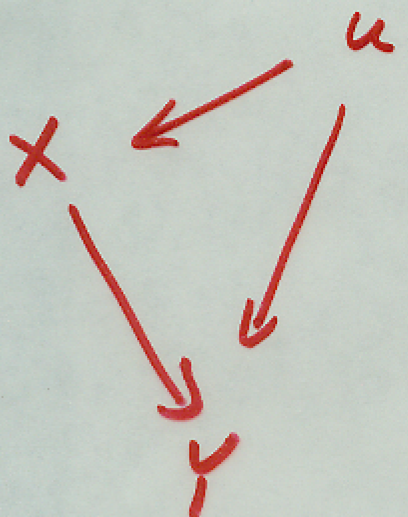


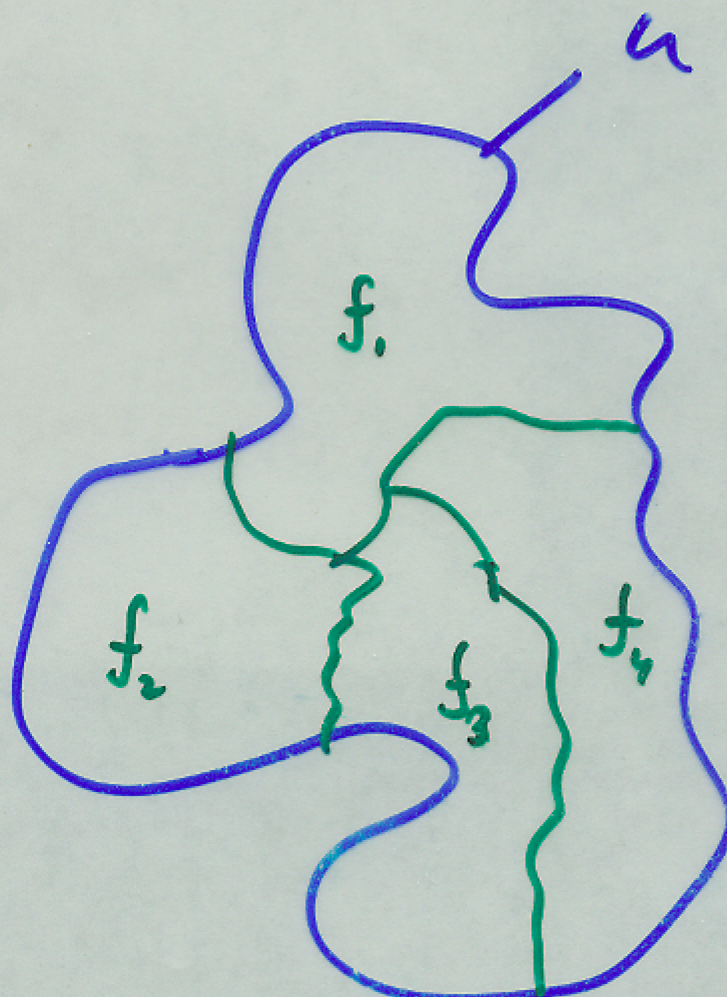
Figure 8.3: A structure equivalent to that of Figure 8.1 but employing finite-state response variables R_z, R_x, and R_y.

Units with deterministic behavior

explained: Equivalence U-classes

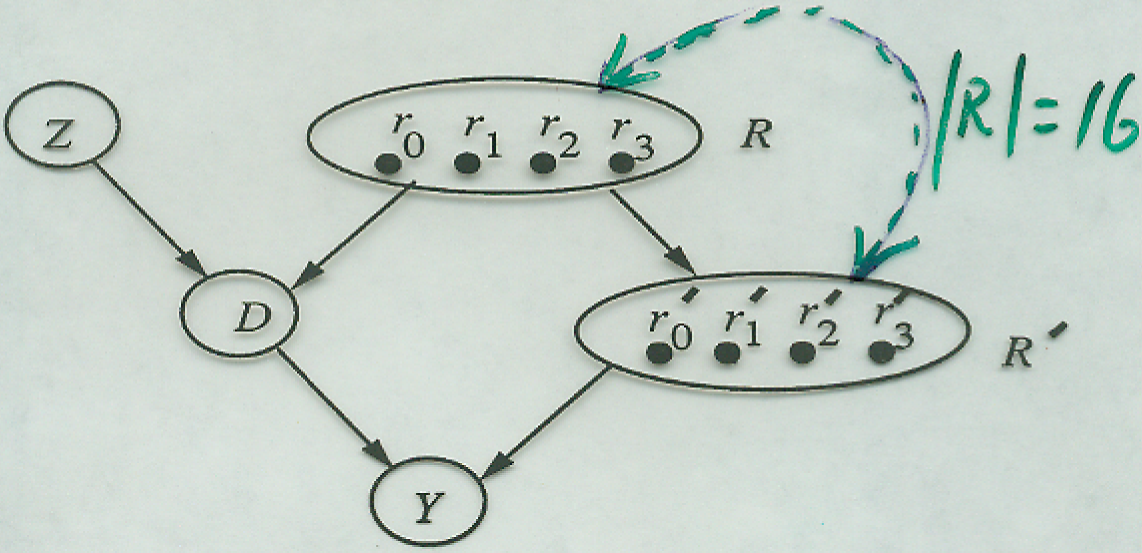


$Y = f(x, u)$



$|\mathrm{Domain}(U)| = 4$

Counterfactual Interpretation of the Latent Variables States



$$y = F_Y(d, r') = \begin{cases} y_0 & \text{if} \quad r' = r'_0 \\[1em] y_0 & \text{if} \quad r' = r'_1 \quad d = d_0 \\ y_1 & \text{if} \quad r' = r'_1 \quad d = d_1 \\[1em] y_1 & \text{if} \quad r' = r'_2 \quad d = d_0 \\ y_0 & \text{if} \quad r' = r'_2 \quad d = d_1 \\[1em] y_1 & \text{if} \quad r' = r'_3 \end{cases}$$

SHARP BOUNDS ON AVERAGE TREATMENT EFFECT (ATE) (BALKE & PEARL, 1993)

(JASA, Sept. 1997)

$$ATE \geq \max \left\{ \begin{array}{c} p_{00.0} + p_{11.1} - 1 \\ p_{00.1} + p_{11.1} - 1 \\ p_{11.0} + p_{00.1} - 1 \\ p_{00.0} + p_{11.0} - 1 \\ 2p_{00.0} + p_{11.0} + p_{10.1} + p_{11.1} - 2 \\ p_{00.0} + 2p_{11.0} + p_{00.1} + p_{01.1} - 2 \\ p_{10.0} + p_{11.0} + 2p_{00.1} + p_{11.1} - 2 \\ p_{00.0} + p_{01.0} + p_{00.1} + 2p_{11.1} - 2 \end{array} \right\}$$

$$ATE \leq \min \left\{ \begin{array}{c} 1 - p_{10.0} - p_{01.1} \\ 1 - p_{01.0} - p_{10.1} \\ 1 - p_{01.0} - p_{10.0} \\ 1 - p_{01.1} - p_{10.1} \\ 2 - 2p_{01.0} - p_{10.0} - p_{10.1} - p_{11.1} \\ 2 - p_{01.0} - 2p_{10.0} - p_{00.1} - p_{01.1} \\ 2 - p_{10.0} - p_{11.0} - 2p_{01.1} - p_{10.1} \\ 2 - p_{00.0} - p_{01.0} - p_{01.1} - 2p_{10.1} \end{array} \right\}$$

holestyramine / several years

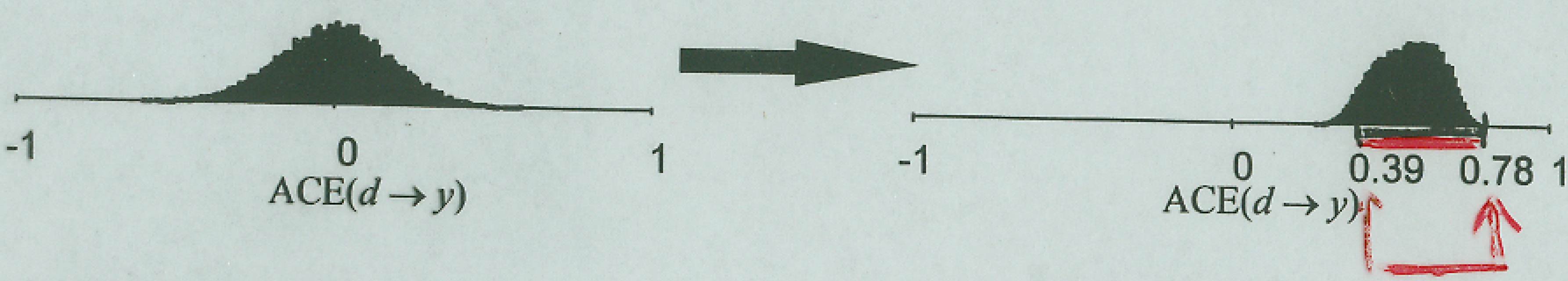
BAYESIAN ESTIMATION OF ACE $(d \to y)$

in

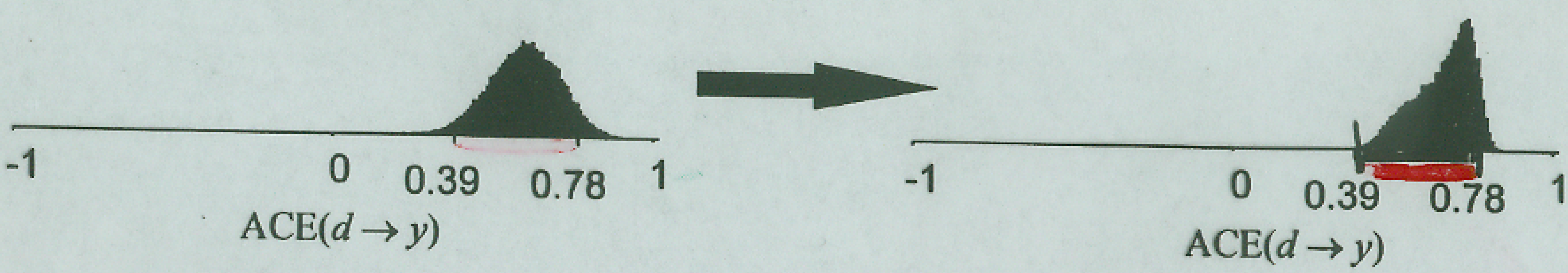
Lipid Study (1984)

337 subjects, 61% compliance [Chickering & Pearl, 1996]

Uninformative (Uniform) Priors



Informative Priors



[Balke & Pearl, 1994]

COUNTERFACTUAL PROBABILITIES: APPLICATIONS TO LIABILITY JUDGMENT

- Joe was given a sample drug (z_1)
  He took the drug (d_1)
  Joe died (y_1)
- Who is responsible?
  The distributor (z_1)? The manufacturer (d_1)?

Given $P(x, y, z)$

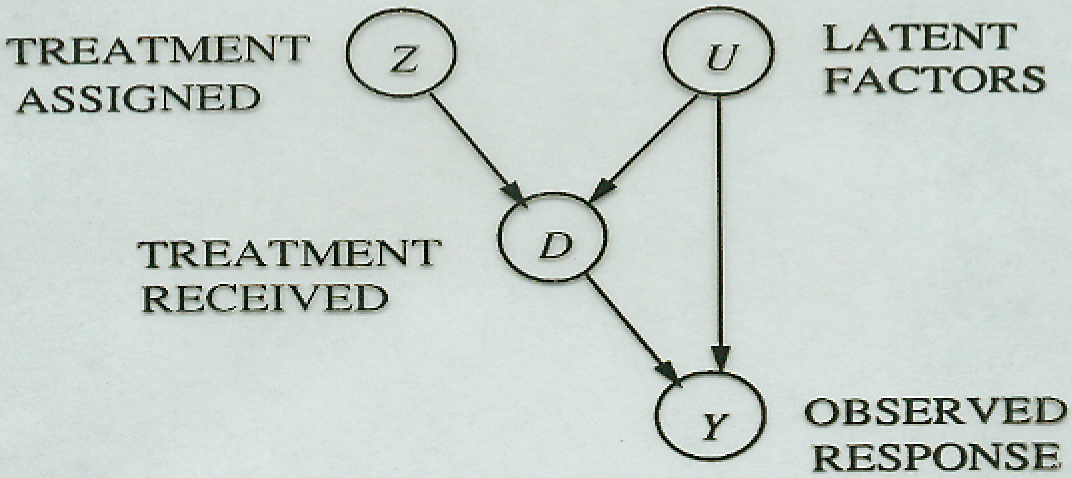
$$\frac{1}{p_{11.1}} \max \left\{ \begin{array}{c} 0 \\ p_{11.1} - p_{00.0} \\ p_{11.0} - p_{00.1} - p_{10.1} \\ p_{10.0} - p_{01.1} - p_{10.1} \end{array} \right\}$$

$$\leq P(Y_{z_0} = y_1 | z_1, d_1, y_1) \leq$$

$$\frac{1}{p_{11.1}} \min \left\{ \begin{array}{c} p_{11.1} \\ p_{10.0} + p_{11.0} \\ 1 - p_{00.0} - p_{10.1} \end{array} \right\}$$

Q_2: Can the model be tested

Necessary and Sufficient Conditions for a Marginal Probability $P(y, d, z)$ to be Generated by the Structure Given in the graph



$$
\begin{aligned}
P(y_1, d_1 | z_1) &\leq 1 - P(y_0, d_1 | z_0) \\
P(y_1, d_1 | z_0) &\leq 1 - P(y_0, d_1 | z_1) \\
P(y_1, d_0 | z_1) &\leq 1 - P(y_0, d_0 | z_0) \\
P(y_1, d_0 | z_0) &\leq 1 - P(y_0, d_0 | z_1)
\end{aligned}
$$

$$\forall d \quad \sum_y \max_z P(y, d | z) \leq 1$$