IDENTIFYING THE EFFECT OF CONDITIONAL ACTIONS

$P(y|do(X = g(z)))$ = the distribution of Y under the policy $do(X = g(z))$.

To compute $P(y|do(X = g(z)))$, we condition on Z and write

$$\begin{aligned} & P(y|do(X = g(z))) \\ = & \sum_z P(y|do(X = g(z)), z) P(z|do(X = g(z))) \\ = & \sum_z P(y|\hat{x}, z)|_{x=g(z)} P(z) \\ = & E_z[P(y|\hat{x}, z)|_{x=g(z)}]. \end{aligned}$$

(using $P(z|do(X = g(z))) = P(z)$)

Conditioning on Z might create dependencies that will prevent the successful reduction of $P(y|\hat{x}, z)$ to a hat-free expression.

IDENTIFYING THE EFFECT OF STOCHASTIC POLICIES

Stochastic policy: enforce the intervention $do(X = x)$ with probability $P^*(x|z)$.

Given $Z = z$, the intervention $do(X = x)$ will occur with probability $P^*(x|z)$ and will produce a causal effect given by $P(y|\hat{x}, z)$. Averaging over x and z gives

$$P(y)|_{P^*(x|z)} = \sum_x \sum_z P(y|\hat{x}, z) P^*(x|z) P(z).$$

$P^*(x|z)$ is specified externally. Therefore, the identifiability of $P(y|\hat{x}, z)$ is a necessary and sufficient condition for the identifiability of any stochastic policy that shapes the distribution of X by the outcome of Z.

IDENTIFYING THE EFFECTS OF PLANS

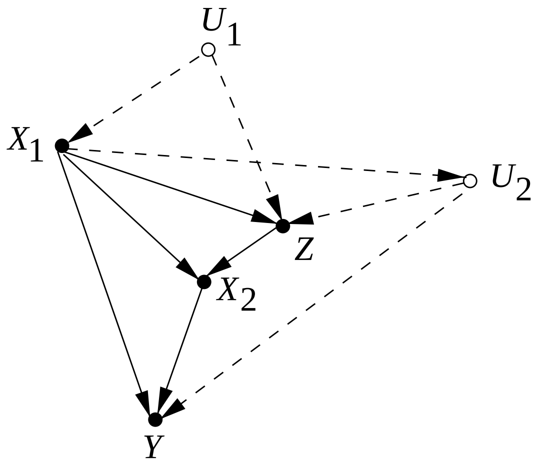
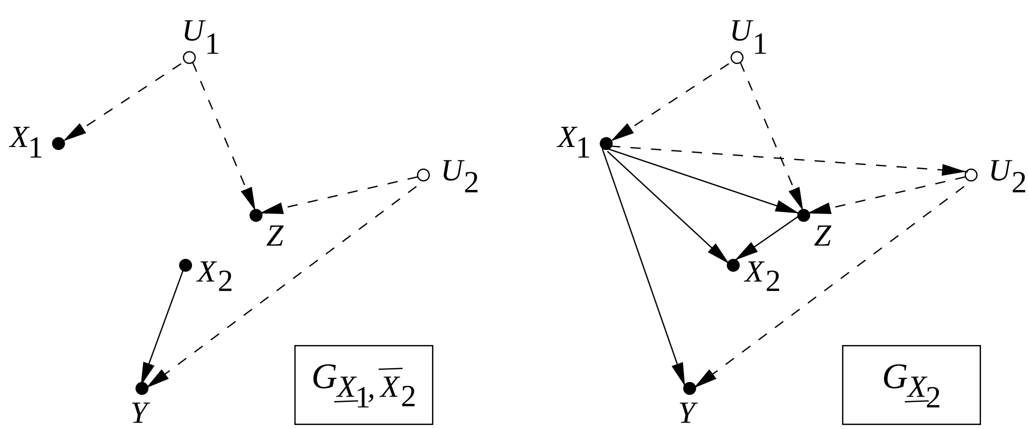


Figure 4.4

$$P(y|\hat{x}_1, \hat{x}_2) = P(y|x_1, \hat{x}_2) \quad \textbf{(4.1)}$$

$$= \sum_z P(y|z, x_1, \hat{x}_2) P(z|x_1) \quad \textbf{(4.2)}$$

$$= \sum_z P(y|z, x_1, x_2) P(z|x_1), \quad \textbf{(4.3)}$$



(a) **Figure 4.5** (b)

PLAN IDENTIFICATION A GENERAL CRITERION

Theorem 4.4.1 (Pearl and Robins 1995)
The probability $P(y|\hat{x}_1, \ldots, \hat{x}_n)$ is identifiable if, for every $1 \leq k \leq n$, there exists a set Z_k of covariates satisfying

$$Z_k \subseteq N_k, \tag{4.4}$$

(i.e., Z_k consists of nondescendants of $\{X_k, X_{k+1}, \ldots, X_n\}$) and

$$(Y \perp\!\!\!\perp X_k | X_1, \ldots, X_{k-1}, Z_1, Z_2, \ldots, Z_k)_{G_{\underline{X}_k, \overline{X}_{k+1}, \ldots, \overline{X}_n}}. \tag{4.5}$$

When these conditions are satisfied, the effect of the plan is given by

$$P(y|\hat{x}_1, \ldots, \hat{x}_n) = \sum_{z_1, \ldots, z_n} P(y|z_1, \ldots, z_n, x_1, \ldots, x_n) \prod_{k=1}^{n} P(z_k|z_1, \ldots, z_{k-1}, x_1, \ldots, x_{k-1}). \tag{4.6}$$

UNFOLDING EQ. 4.5

$$(Y \perp\!\!\!\perp X_k | X_1, \ldots, X_{k-1}, Z_1, Z_2, \ldots, Z_k)_{G_{\underline{X}_k, \overline{X}_{k+1}, \ldots, \overline{X}_n}}. \quad \textbf{(4.5)}$$

$$(Y \perp\!\!\!\perp X_1 | Z_1)_{G_{\underline{X}_1, \overline{X}_2, \overline{X}_3, \ldots, \overline{X}_n}} \qquad k = 1$$

$$(Y \perp\!\!\!\perp X_2 | X_1, Z_1, Z_2)_{G_{\underline{X}_2, \overline{X}_3, \ldots, \overline{X}_n}} \qquad k = 2$$

PLAN IDENTIFICATION A PROCEDURE

Theorem 4.4.6
The probability $P(y|\hat{x}_1,\ldots,\hat{x}_n)$ is G-identifiable if and only if the following condition holds for every $1 \leq k \leq n$:

$$(Y \perp\!\!\!\perp X_k | X_1,\ldots,X_{k-1},W_1,W_2,\ldots,W_k)_{G_{\underline{X}_k,\overline{X}_{k+1},\ldots,\overline{X}_n}},$$

where W_k is the set of all covariates in G that are both nondescendants of $\{X_k,X_{k+1},\ldots,X_n\}$ and have either Y or X_k as descendant in $G_{\underline{X}_k,\overline{X}_{k+1},\ldots,\overline{X}_n}$. Moreover, if this condition is satisfied then the plan evaluates as

$$P(y|\hat{x}_1,\ldots,\hat{x}_n) = \sum_{w_1,\ldots,w_n} P(y|w_1,\ldots,w_n,x_1,\ldots,x_n) \prod_{k=1}^{n} P(w_k|w_1,\ldots,w_{k-1},x_1,\ldots,x_{k-1}). \quad \textbf{(4.8)}$$

DIRECT EFFECTS

Definition 4.5.1 (Direct Effect)
The direct effect of X on Y is given by $P(y|\hat{x}, \hat{s}_{XY})$, where S_{XY} is the set of all endogenous variables except X and Y in the system.

Corollary 4.5.2
The direct effect of X on Y is given by $P(y|\hat{x}, \widehat{pa}_{Y\backslash X})$, where $pa_{Y\backslash X}$ stands for any realization of the parents of Y, excluding X.

Theorem 4.5.3
Let $PA_Y = \{X_1, \ldots, X_k, \ldots, X_m\}$. The direct effect of any X_k on Y is identifiable whenever the conditions of Corollary 4.4.5 hold for the plan $(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_m)$ in some admissible ordering of the variables. The direct effect is then given by (4.8).

Corollary 4.5.4
Let X_j be a parent of Y. The direct effect of X_j on Y is, in general, nonidentifiable if there exists a confounding arc that embraces any link $X_k \to Y$.

EXAMPLE: SEX DISCRIMINATION

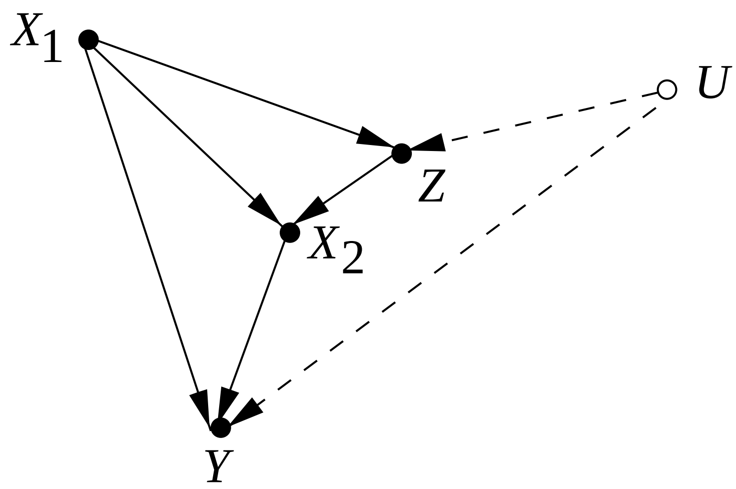


Figure 4.9

$X_1 =$ applicant's gender;
$X_2 =$ applicant's choice of department;
$Z =$ applicant's career objectives;
$Y =$ admission outcome (accept/reject);
$U =$ applicant's aptitude (unrecorded).
Adjusting for department choice gives:

$$E_{x_2} P(y|\hat{x}_1, x_2) = \sum_{x_2} P(y|x_1, x_2) P(x_2). \quad \textbf{(4.9)}$$

while the direct effect of X_1 on Y, as given by (4.7), reads

$$P(y|\hat{x}_1, \hat{x}_2) = \sum_{z} P(y|z, x_1, x_2) P(z|x_1). \quad \textbf{(4.10)}$$