CAUSAL STRUCTURE

Definition 2.2.1 (Causal Structure)
A causal structure of a set of variables V is a directed acyclic graph **(DAG)** in which each node corresponds to a distinct element of V, and each link represents direct functional relationship among the corresponding variables.

Definition 2.2.2 (Causal Model)
A causal model is a pair $M = <D, \Theta_D>$ consisting of a causal structure D and a set of parameters Θ_D compatible with D. The parameters Θ_D assign a function $x_i = f_i(pa_i, u_i)$ to each $X_i \in V$ and a probability measure $P(u_i)$ to each u_i, where PA_i are the parents of X_i in D and where each U_i is a random disturbance distributed according to $P(u_i)$, independently of all other u.

Definition 2.3.1 (Inferred Causation (Preliminary))
A variable X is said to have a **causal influence** on a variable Y if a directed path from X to Y exists in every minimal structure consistent with the data.

LATENT STRUCTURE

Definition 2.3.2 (Latent Structure)
A **latent structure** is a pair $L = <D, O>$, where D is a causal structure over V and where $O \subseteq V$ is a set of observed variables.

Definition 2.3.3 (Structure Preference)
One latent structure $L = <D, O>$ is **preferred** to another $L' = <D', O>$ (written $L \preceq L'$) if and only if D' can mimic D over O—that is, if and only if for every Θ_D there exists a $\Theta'_{D'}$ such that $P_{[O]}(<D', \Theta'_{D'}>) = P_{[O]}(<D, \Theta_D>)$. Two latent structures are **equivalent**, written $L' \equiv L$, if and only if $L \preceq L'$ and $L \succeq L'$.

Definition 2.3.4 (Minimality)
A latent structure L is **minimal** with respect to a class $\mathcal{L}$ of latent structures if and only if there is no member of $\mathcal{L}$ that is strictly preferred to L—that is, if and only if for every $L' \in \mathcal{L}$ we have $L \equiv L'$ whenever $L' \preceq L$.

Definition 2.3.5 (Consistency)
A latent structure $L = <D, O>$ is **consistent** with a distribution $\hat{P}$ over O if D can accommodate some model that generates $\hat{P}$—that is, if there exists a parameterization Θ_D such that $P_{[O]}(<D, \Theta_D>) = \hat{P}$.

Definition 2.3.6 (Inferred Causation)
Given $\hat{P}$, a variable C has a **causal influence** on variable E if and only if there exists a directed path from C to E in every minimal latent structure consistent with $\hat{P}$.

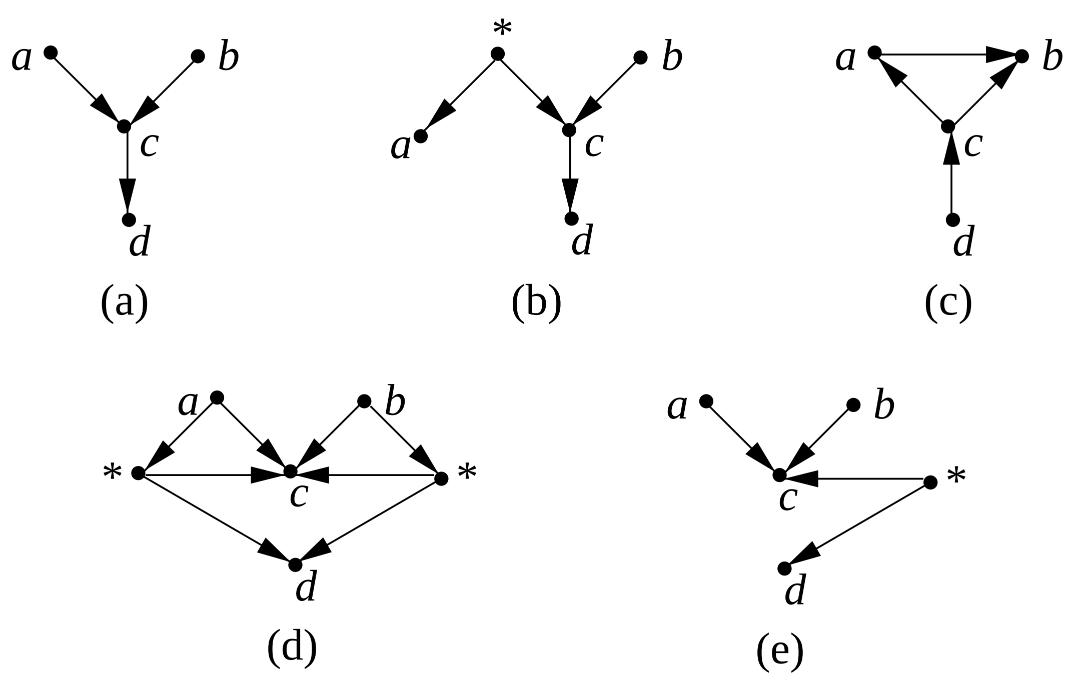

Figure 2.1

STABILITY

Definition 2.4.1 (Stability)
Let $I(P)$ denote the set of all conditional independence relationships embodied in P. A causal model $M = <D, \Theta_D>$ generates a stable distribution if and only if $P(<D, \Theta_D>)$ contains no extraneous independences—that is, if and only if $I(P(<D, \Theta_D>)) \subseteq I(P(<D, \Theta'_D>))$ for any set of parameters Θ'_D.

Definition 2.6.1 (Projection)
A latent structure $L_{[O]} = <D_{[O]}, O>$ is a **projection** of another latent structure L if and only if:

1. every unobservable variable of $D_{[O]}$ is a parentless common cause of exactly two nonadjacent observable variables.
2. for every stable distribution P generated by L, there exists a stable distribution P' generated by $L_{[O]}$ such that $I(P_{[O]}) = I(P'_{[O]})$.

INDUCTIVE CAUSATION

IC Algorithm (Inductive Causation)

Input: $\widehat{P}$, a stable distribution on a set V of variables.

Output: a pattern $H(\widehat{P})$ compatible with $\widehat{P}$.

1. For each pair of variables a and b in V, search for a set S_{ab} such that $(a \perp\!\!\!\perp b | S_{ab})$ holds in $\widehat{P}$—in other words, a and b should be independent in $\widehat{P}$, conditioned on S_{ab}. Construct an undirected graph G such that vertices a and b are connected with an edge if and only if no set S_{ab} can be found.
2. For each pair of nonadjacent variables a and b with a common neighbor c, check if $c \in S_{ab}$.
   If it is, then continue.
   If it is not, then add arrowheads pointing at c (i.e., $a \rightarrow c \leftarrow b$).
3. In the partially directed graph that results, orient as many of the undirected edges as possible subject to two conditions: (i) the orientation should not create a new v-structure; and (ii) the orientation should not create a directed cycle.

RULES FOR ORIENTING EDGES

R_1: Orient b–c into $b \to c$ whenever there is an arrow $a \to b$ such that a and c are non adjacent.

R_2: Orient a–b into $a \to b$ whenever there is chain $a \to c \to b$.

R_3: Orient a–b into $a \to b$ whenever there are two chains a–$c \to b$ and a—$d \to b$ such that c and d are nonadjacent.

R_4: Orient a–b into $a \to b$ whenever there are two chains a–$c \to d$ and $c \to d \to b$ such that c and b are nonadjacent.

INDUCTIVE CAUSATION WITH LATENT VARIABLES

IC* Algorithm (Inductive Causation with Latent Variables)

Input: $\hat{P}$, a sampled distribution.

Output: $core(\hat{P})$, a marked pattern.

1. For each pair of variables a and b, search for a set S_{ab} such that a and b are independent in $\hat{P}$, conditioned on S_{ab}.
   If there is no such S_{ab}, place an undirected link between the two variables, $a - b$.

2. For each pair of nonadjacent variables a and b with a common neighbor c, check if $c \in S_{ab}$.
   If it is, then continue.
   If it is not, then add arrowheads pointing at c (i.e., $a \rightarrow c \leftarrow b$).

3. In the partially directed graph that results, add (recursively) as many arrowheads as possible, and mark as many edges as possible, according to the following two rules:

   - R_1: For each pair of non-adjacent nodes a and b with a common neighbor c, if the link between a and c has an arrowhead into c and if the link between c and b has no arrowhead into c, then add an arrowhead on the link between c and b pointing at b and mark that link to obtain $c \overset{*}{\rightarrow} b$.

   - R_2: If a and b are adjacent and there is a directed path (composed strictly of marked links) from a to b (as in Figure 2.2), then add an arrowhead pointing toward b on the link between a and b.

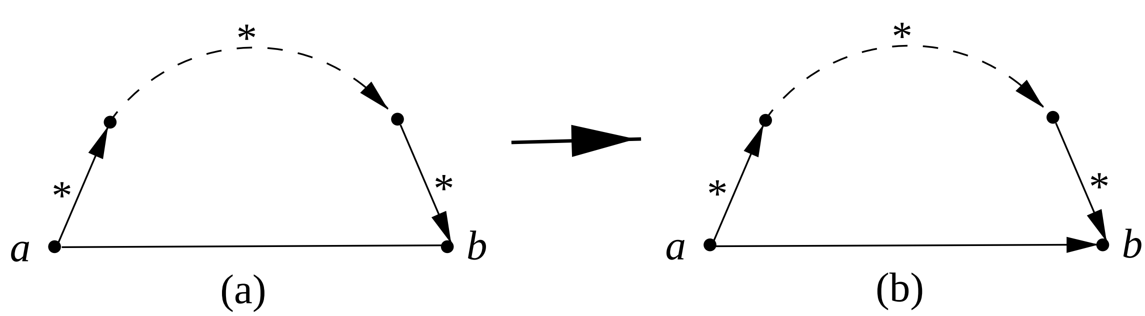

Figure 2.2

EXAMPLE

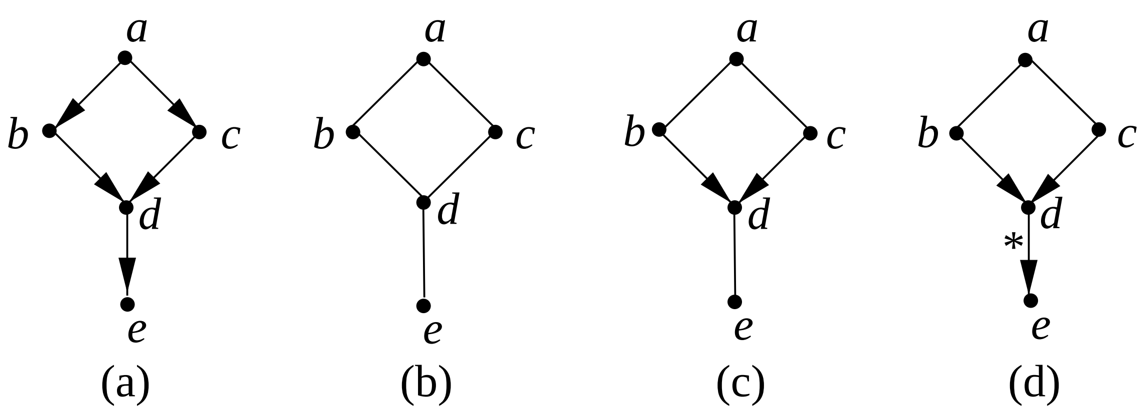

Figure 2.3

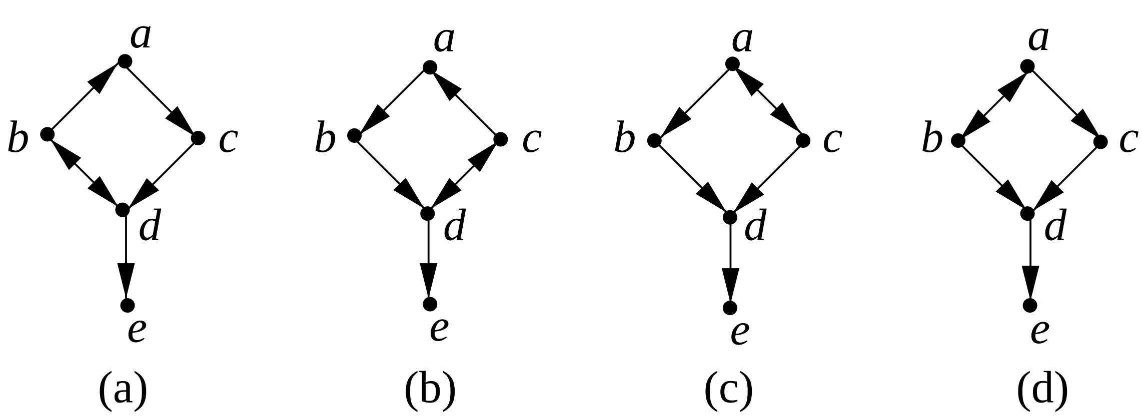

Figure 2.4

LOCAL CONDITIONS FOR CAUSATION

Definition 2.7.1 (Potential Cause)
A variable X has a **potential causal influence** on another variable Y (that is **inferable** from $\hat{P}$) if the following conditions hold.

1. X and Y are dependent in every context.
2. There exists a variable Z and a context S such that

   (i) X and Z are independent given S (i.e., $X \perp\!\!\!\perp Z | S$) and

   (ii) Z and Y are dependent given S (i.e., $Z \not\perp\!\!\!\perp Y | S$).

GENUINE CAUSE

Definition 2.7.2 (Genuine Cause)
A variable X has a **genuine causal influence** on another variable Y if there exists a variable Z such that either:

1. X and Y are dependent in any context and there exists a context S satisfying

   (i) Z is a potential cause of X (per Definition 2.7.1),

   (ii) Z and Y are dependent given S (i.e., $Z \not\perp\!\!\!\perp Y | S$), and

   (iii) Z and Y are independent given $S \cup X$ (i.e., $Z \perp\!\!\!\perp Y | S \cup X$);

   or

2. X and Y are in the transitive closure of the relation defined in criterion 1.

SPURIOUS ASSOCIATION

Definition 2.7.3 (Spurious Association)
Two variables X and Y are spuriously associated if they are dependent in some context and there exist two other variables (Z_1 and Z_2), and two contexts (S_1 and S_2), such that:

1. Z_1 and X are dependent given S_1 (i.e., $Z_1 \not\perp\!\!\!\perp X | S_1$);
2. Z_1 and Y are independent given S_1 (i.e., $Z_1 \perp\!\!\!\perp Y | S_1$);
3. Z_2 and Y are dependent given S_2 (i.e., $Z_2 \not\perp\!\!\!\perp Y | S_2$); and
4. Z_2 and X are independent given S_2 (i.e., $Z_2 \perp\!\!\!\perp X | S_2$).

Definition 2.7.4 (Genuine Causation with Temporal Information)
A variable X has a causal influence on Y if there is a third variable Z and a context S, both occurring before X, such that:

1. $(Z \not\perp\!\!\!\perp Y | S)$;
2. $(Z \perp\!\!\!\perp Y | S \cup X)$.

SPURIOUS ASSOCIATION WITH TEMPORAL INFORMATION

Definition 2.7.5 (Spurious Association with Temporal Information)
Two variables X and Y are **spuriously associated** if they are dependent in some context S, if X precedes Y, and if there exists a variable Z satisfying:

1. $(Z \perp\!\!\!\perp Y | S)$;
2. $(Z \not\perp\!\!\!\perp X | S)$.

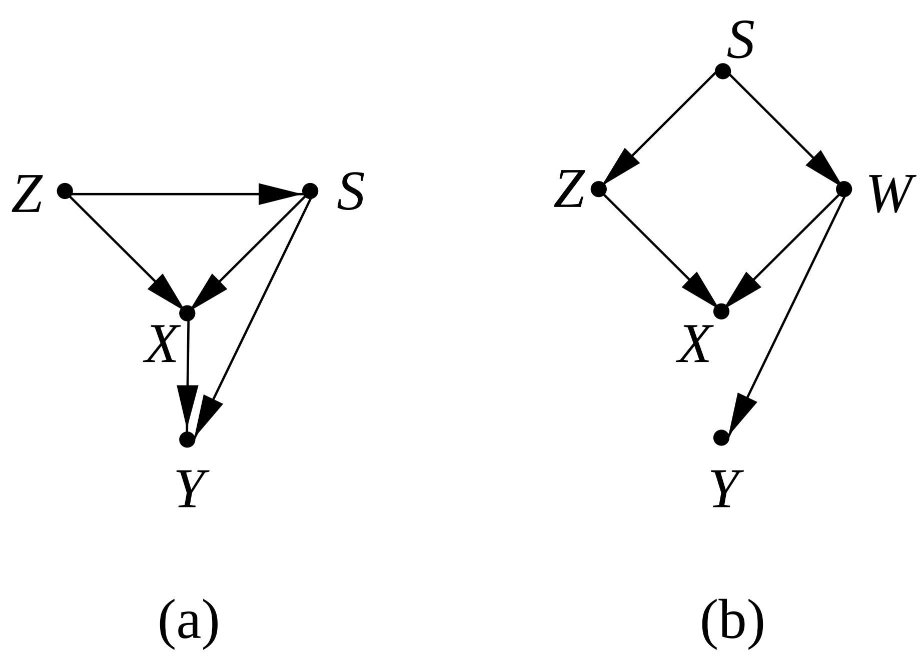

Figure 2.5

STATISTICAL TIME

Definition 2.8.1 (Statistical Time)
Given an empirical distribution P, a **statistical time** of P is any ordering of the variables that agrees with at least one minimal causal structure consistent with P.

Conjecture 2.8.2 (Temporal Bias)
In most natural phenomenon, the physical time coincides with at least one statistical time.

MARKOV-EQUIVALENT MODELS

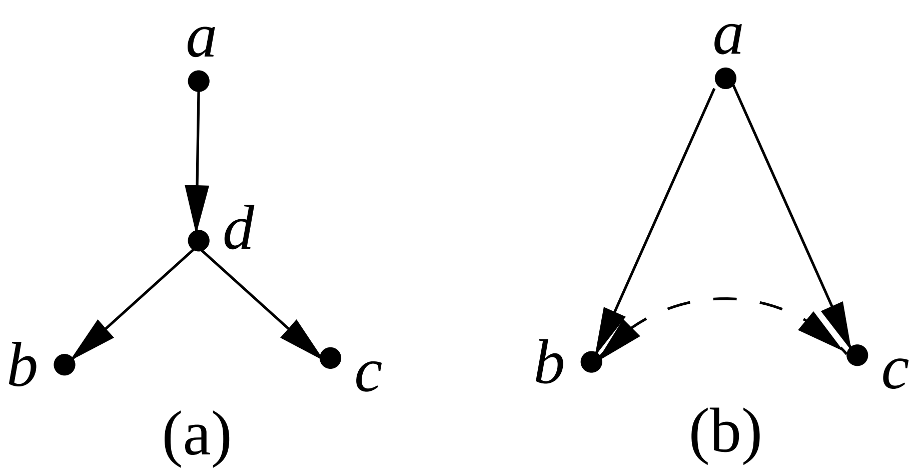

Figure 2.6