

Causal Inference in Statistics:
A Primer
Sample of Solution Manual

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About This Sample

This document provides (a sample of the full manual's) solutions, explanations, and intuition for the study questions posed in *Causality in Statistics: A Primer*. Students are encouraged to attempt each study question by hand before consulting the answers herein.

Obtaining the Full Manual

The full solution manual is only available to instructors considering, or already using, the text. To request access to the full solution manual, you can register as an instructor with the publisher, Wiley, using the following steps:

- Navigate to:
<http://www.wiley.com/WileyCDA/WileyTitle/productCd-1119186846.html>
- At the bottom of the page, click on the button reading "Visit Companion Site."
- On the companion page's navigation bar, click on "Browse by Resource" and then "Solution to study questions"
- On the next page, click the "Solution to study questions" link, at which point you will need to have a verified Wiley instructor account to view the full manual.

If, for whatever reason, you encounter difficulties accessing the manual through Wiley's registration process, you may contact the authors by sending an email to causality.ucla@gmail.com explaining the situation.

Online Access

As the authors make updates to the text and solution manual, changes and errata will be posted at the following links:

Textbook Information & Update site: <http://bayes.cs.ucla.edu/PRIMER/>
Solution Manual Information & Update site: <http://bayes.cs.ucla.edu/PRIMER/Manual>

Interactive Tutorial using DAGitty

The authors have collaborated with Johannes Textor, the maker of DAGitty (a browser-based environment for creating, editing, and analyzing causal models), to provide interactive tutorials for classroom use and self-study. We provide solutions to some exercises in the *R* environment for statistical computing, based on the DAGitty *R* package. Each question with an accompanying DAGitty explanation has been tagged accordingly, and students may find a complete list of these examples at the following link:

[DAGitty] Textbook Companion site: <http://dagitty.net/primer/>

Study Questions and Solutions for Chapter 1

Study question 1.2.1 (Monty Hall).

Prove, using Bayes' theorem, that switching doors improves your chances of winning the car in the Monty Hall problem.

Solution to study question 1.2.1

Let X indicate the door chosen by the player, Y indicate the door hiding the car, and Z indicate the door opened by the host. We want to prove that:

$$P(Y = A|X = A, Z = C) < P(Y = B|X = A, Z = C)$$

So, we'll compute the components of this expression necessary to illustrate this inequality, and then combine them. We see that switching doors doubles our chances of winning.

$$\begin{aligned}P(Z = C|X = A) &= P(Z = C|X = A, Y = A)P(Y = A) \\ &\quad + P(Z = C|X = A, Y = B)P(Y = B) \\ &\quad + P(Z = C|X = A, Y = C)P(Y = C) \\ &= 1/2 * 1/3 + 1 * 1/3 + 0 * 1/3 \\ &= 1/2\end{aligned}$$

$$\begin{aligned}P(Y = A|X = A, Z = C) &= \frac{P(Z = C|X = A, Y = A)P(Y = A|X = A)}{P(Z = C|X = A)} \\ &= \frac{1/2 * 1/3}{1/2} \\ &= 1/3\end{aligned}$$

$$\begin{aligned}P(Y = B|X = A, Z = C) &= 1 - P(Y = A|X = A, Z = C) - P(Y = C|X = A, Z = C) \\ &= 1 - 1/3 - 0 \\ &= 2/3\end{aligned}$$

Study Questions and Solutions for Chapter 2

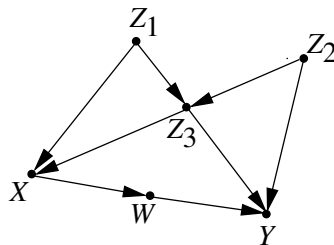


Figure 2.5: A causal graph used in study question 2.5.1, all U terms (not shown) are assumed independent

Study question 2.5.1.

(a) Which of the arrows in Figure 2.5 can be reversed without being detected by any statistical test? [Hint: Use the criterion for equivalence class.]

Solution to study question 2.5.1

An R solution of this exercise is provided at dagitty.net/primer/2.5.1

Part (a)

To determine which arrows can be reversed without being detectable by a statistical test, we consider models that are in the *equivalence class* of Figure 2.9 (see paragraphs previous to these study questions in the text). Accordingly, we first find the v -structures in the graph (i.e., colliders whose parents are not adjacent), which are:

- $Z_1 \rightarrow Z_3 \leftarrow Z_2$
- $Z_3 \rightarrow Y \leftarrow W$
- $Z_2 \rightarrow Y \leftarrow W$

So, to find other models in this equivalence class, we may flip the direction of edges such that the resulting model abides by two criteria:

- We neither create nor destroy any v -structures.
- We must not introduce a cycle into the resulting graph. Note that while d -separation is valid in linear cyclical models, it is not valid in general, namely, for any non-linear functions.

With these constraints, we conclude that there are *no such edges* that can be reversed within the model to find another within its equivalence class. We would verify this claim by testing our constraints against each edge reversal. Here is a complete list of our tests and a reason (of which there might be several) why each reversal fails to produce a model in the equivalence class of Figure 2.9 (note, we define the “old model” as the one pre-edge-reversal, and the “new model” as the one resulting from the reversal):

Edge	Reason
$Z_1 \rightarrow X$	Creates a cycle in new model, $Z_1 \rightarrow Z_3 \rightarrow X \rightarrow Z_1$
$Z_1 \rightarrow Z_3$	Destroys a v -structure in old model, $Z_1 \rightarrow Z_3 \leftarrow Z_2$
$Z_3 \rightarrow X$	Creates a v -structure in new model, $X \rightarrow Z_3 \leftarrow Z_2$
$Z_3 \rightarrow Y$	Creates a v -structure in new model, $Y \rightarrow Z_3 \leftarrow Z_1$
$X \rightarrow W$	Creates a v -structure in new model, $W \rightarrow X \leftarrow Z_1$
$W \rightarrow Y$	Destroys a v -structure in old model, $W \rightarrow Y \leftarrow Z_2$
$Z_2 \rightarrow Z_3$	Destroys a v -structure in old model, $Z_1 \rightarrow Z_3 \leftarrow Z_2$
$Z_2 \rightarrow Y$	Destroys a v -structure in old model, $Z_2 \rightarrow Y \leftarrow W$

Study Questions and Solutions for Chapter 3

Study question 3.3.1.

Consider the graph in Figure 3.8:

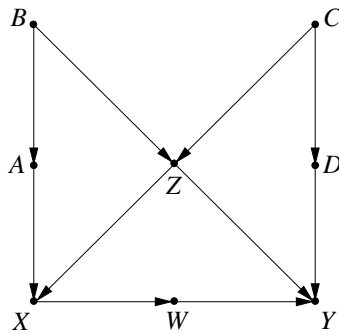


Figure 3.8: Causal graph used to illustrate the backdoor criterion in the following study questions

- (a) List all of the sets of variables that satisfy the backdoor criterion to determine the causal effect of X on Y .

Solution to study question 3.3.1

The graph of this exercise is available at dagitty.net/m331. Students can interactively modify this graph and see the implications for the backdoor adjustment sets.

An R solution of this exercise is provided at dagitty.net/primer/3.3.1

Part (a)

Let us use the abbreviation "backdoor admissible" to denote a set of variables that satisfy the backdoor criterion of Definition 3.3.1; for the present model, a backdoor admissible set Z blocks all spurious paths between X and Y while leaving all directed paths from X to Y open. We can easily verify that the following sets satisfy the backdoor criterion to determine the causal effect of X on Y .

1. *Sets of 2 nodes:* $\{Z, A\}, \{Z, B\}, \{Z, C\}, \{Z, D\}$
2. *Sets of 3 nodes:* $\{Z, A, B\}, \{Z, A, C\}, \{Z, A, D\}, \{Z, B, C\}, \{Z, B, D\}, \{Z, C, D\}$
3. *Sets of 4 nodes:* $\{Z, A, B, C\}, \{Z, A, B, D\}, \{Z, A, C, D\}, \{Z, B, C, D\}$
4. *Sets of 5 nodes:* $\{Z, A, B, C, D\}$

Study Questions and Solutions for Chapter 4

Study question 4.3.1.

(a) Prove that, if X is binary, the effect of treatment on the treated can be estimated from both observational and experimental data. Hint: Decompose $E[Y_x]$ into

$$E[Y_x] = E[Y_x|x']P(x') + E[Y_x|x]P(x)$$

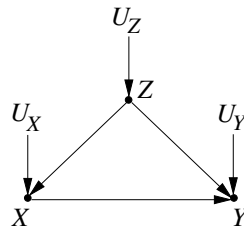


Figure 3.3: A graphical model representing the effects of a new drug, with Z representing gender, X standing for drug usage, and Y standing for recovery

Solution to study question 4.3.1

Part (a)

We begin by noting that, since X is binary, we can simply use the law of total probability and write:

$$E[Y_x] = E[Y_x|X = x]P(X = x) + E[Y_x|X = x']P(X = x')$$

By the *consistency* axiom, we also know that:

$$E[Y_x|X = x] = E[Y|X = x]$$

$$\therefore E[Y_x] = E[Y|X = x]P(X = x) + E[Y_x|X = x']P(X = x')$$

The consistency axiom intuitively follows from the notion that a counterfactual predicated on an actual observation is not counterfactual (here we observed $X = x$ and were hypothesizing about Y_x). The term $E[Y|X = x]P(X = x)$ is already estimable from observational data, so it remains to address the other. Solving for $E[Y_x|X = x']$, gives:

$$E[Y_x|X = x'] = \frac{E[Y_x] - E[Y|X = x]P(X = x)}{P(X = x')}$$

Now, substituting back into our equation for the ETT:

$$\begin{aligned} ETT &= E[Y_x - Y_{x'}|X = x] \\ &= E[Y|X = x] - E[Y_{x'}|X = x] \\ &= E[Y|X = x] - \frac{E[Y|do(X = x')] - E[Y|X = x']P(X = x')}{P(X = x)} \end{aligned}$$

We see that the effect of treatment on the treated can be estimated from non-experimental (*do*-free expressions) and experimental data (expressions with the *do*-operator).