CS262C HOMEWORK #6

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Question 1

You and two partners meet in a room to coordinate your future actions. You know that after you separate, each of you will be asked one of two questions: X or Y. To each question you must answer either $+1$ or -1, without communicating with your partners.

The task is to devise a question-answering strategy, one for each partner, that results in the following behavior:

(i) whenever two of the partners are asked a Y question, the number of $+1$ answers must be even.

(ii) whenever all three partners are asked a X question, the number of $+1$ answers must be odd.

1.a Show, using our counterfactual notation, that no such strategy exists.

1.b How would the answer change if we allow the partners to use stochastic strategies, namely, each partner determines his answer after consulting a random device, and the three random devices are correlated.

Question 2

In an employment-discrimination case known as Carson versus Bethlehem Steel Corp. (70 FEP Cases 921, 7th Cir. (1996)) the court wrote:

"The central question in any employment-discrimination case is whether the employer would have taken the same action had the employee been of different race (age, sex, religion, national origin, etc.) and everything else had been the same."

Let X be the employee sex ($x = $ male, $x' = $ female), Y the employer action ($y = $ hire, $y' = $ not hire) and Z stands for "everything else", namely,

all other variables in V, including gender-dependent variables such as skills and qualifications.

2.a Express this interpretation of the anti-discrimination ruling as a formal counterfactual assertion.

2.b Write a counterfactual expression for the probability that the ruling above would be violated in a probabilistic model $\langle M, P(u) \rangle$.

2.c Assume that the person in question is a female who was not hired and has characteristics $Z = z$. Write an expression for the probability that she would have been hired, had she been a male and everything else (i.e., Z) had been the same.

2.d Can the probability above (question 2.c) be written as an expression in do-calculus?

2.e Write an expression for the increase in female hiring of forcing all employers to comply with the anti-discrimination rule and treat female applications the same as male's.

2.f Under what conditions can the increase above (question 2.e) be written as an expression in do-calculus?

2.g Under what conditions can the increase above (question 2.e) be identified from the joint probability $P(x, y, z)$?

Question 3

Write an expression for the probability that "I will get cancer if I smoke, given that I was about to start smoking". Show that this probability is identifiable whenever the causal effect (of smoking on cancer) is identifiable.