Homework # 1
The deterministic reading of structural equations

Consider the following structural model:

$$\begin{aligned} W &= e_0 \\ X &= aW + e_1 \\ Z &= bX + cW + e_2 \\ Y &= dX + tZ + e_3 \end{aligned}$$

Where X, Y, Z and W are variables, $e_0, \ldots, e_3$ are "error terms" (or "disturbances"), and a, b, c, d and t are KNOWN and CORRECT structural coefficients.

1. Express the value of X as a function of all error terms.

2. Suppose we measure the values of all error terms, e_0, e_1, e_2, e_3, what value should we observe for Y?

3. Suppose we observe that $X = 1$; what does it tell us about the values of the error terms, at the time of observation.

4. (a) Suppose we observe that $X = 1$; what does it tell us about the values of the other variables, assuming that the errors are unknown but equal, $e_0 = e_2 = e_2 = e_3$.

   (b) Suppose we intervene and set the value of X to $X = 1$, what does it tell us about the values of the other variables, assuming the errors are unknown but equal, $e_0 = e_2 = e_2 = e_3$.

   (c) Suppose we intervene and set the value of X to $X = 2$ immediately after the observation of question 4(a). What will be the new values of W, Z, and Y?

5. Find the values that Y, W, and Z would obtain in an experiment where X is observed to have the value x. Express the answer as a function of x, e_0, e_1, e_2 and e_3,

6. Find the values that Y, W, and Z would obtain in an experiment where X is physically held constant at $X = x$. Express the answer as a function of x, e_0, e_1, e_2 and e_3,

7. Repeat the experiment of question 5 with $X = x + 1$, instead of $X = x$, and record the change in all variables. (Assume the error terms do not change between the two interventions).

8. Suppose we do not know the error terms, but we know that they stay constant between two successive experiments. In the first experiment, we measure $X = 1$. In the second, we intervene and set X to $X = 2$. Compute the predicted change in all variables.

9. Repeat Question 8 with the following two experiments: First we intervene and set X to $X = 1$, then we let go of X and measure its value to be $X = 2$. Compute the predicted change in all variables.

10. Repeat Question 8 with the following two experiments: First we intervene and set the values of X to $X = 1$. Second, we change the setting of X, to $X = 2$ while holding Z constant at whatever value it had in the first experiment. Compute the predicted changes in all variables.

11. Repeat Question 8 with the following two experiments: First we intervene and set the values of X to $X = 1$. Second, we keep the value of X unchanged, at $X = 1$, but we intervene and set the value of Z to whatever value Z would attain if we were to set X to $X = 2$, instead of $X = 1$. Compute the predicted change in all variables.

12. Compare the changes computed for Y in problems 8, 10 and 11, and relate them to your conception of total, direct, and indirect effects. Use these finding to formulate a general definition of total, direct, and indirect effects.

13. Consider the reciprocal model

$$\begin{aligned} x &= by + e_1 \\ y &= cx + e_2 \end{aligned}$$

(a) Compute the total, direct, and indirect effects of x on y according to standard textbooks (e.g., Bollen 1989, p.376)

(b) Compute the total, direct, and indirect effects of x on y according to the definition you formulated in question 12

(c) Discuss the discrepancies (if any) between the 13(a) and 13(b). Can you formulate empirical definitions for 13(a)?

14. Let Y_x be the value that Y would obtain in the experiment of Question 6. Construct another model, M', such that, for all $e_0, e_1, e2$ and e_3, the solution for Y in M' would equal Y_x.

15. (Generalizing question 14) Let X and Y be any two variables in a structural model M, and let Y_x be the value that Y would obtain (according to M) in an experiment where X is held constant at x. Construct another model, M', on the same set of variables, such that, for any Y, the solution of Y in M' will be equal to Y_x.