# Chapter 7

# THE LOGIC OF STRUCTURE-BASED COUNTERFACTUALS

*And the Lord said,*
*"If I find in the city of Sodom fifty good men,*
*I will pardon the whole place for their sake."*
Genesis 18:26

## Preface

This chapter provides a formal analysis of structure-based *counterfactuals*, a concept introduced briefly in Chapter 1 that will occupy the rest of our discussion in this book. Through this analysis, we will obtain sharper mathematical definitions of other concepts that were introduced in earlier chapters, including causal models, action, causal effects, causal relevance, error terms, and exogeneity.

After casting the concepts of causal model and counterfactuals in abstract mathematical terms we will demonstrate by examples how counterfactual questions can be answered from both deterministic and probabilistic causal models (Section 7.1). In Section 7.2.1, we will argue that policy analysis is an exercise in counterfactual reasoning and demonstrate this thesis in a simple example taken from econometrics.

This will set the stage for our discussion in Section 7.2.2, where we explicate the empirical content of counterfactuals in terms of policy predictions. Section 7.2.3 discusses the role of counterfactuals in the interpretation and generation of causal explanations. Section 7.2 concludes with discussions of how causal relationships emerge from actions and mechanisms (Section 7.2.4) and how causal directionality can be induced from a set of symmetric equations (Section 7.2.5).

Section 7.3 we develop an axiomatic characterization of counterfactual and causal relevance relationships as they emerge from the structural model semantics. Section 7.3.1 will identify a set of properties, or axioms, that allow us to derive new counterfactual relations from assumptions, and Section 7.3.2 demonstrates the use of these axioms in algebraic derivation of causal effects. Section 7.3.3 introduces axioms for the relationship of causal relevance and, using their similarity to the axioms of graphs, describes the use of graphs for verifying relevance relationships.

The axiomatic characterization developed in Section 7.3 enables us to compare structural models with other approaches to causality and counterfactuals, most notably those based on Lewis's closest-world semantics (Sections 7.4.1–7.4.4). The formal equivalence of the structural approach and the Neyman-Rubin potential-outcome framework is discussed in Section 7.4.4. Finally, we will revisit the topic of exogeneity and will extend our discussion of Section 5.4.3 with counterfactual definitions of exogenous and instrumental variables in Section 7.4.5.

The final part of this chapter (Section 7.5) compares the structural account of causality with that based on probabilistic relationships. We elaborate our preference toward the structural account and highlight the difficulties that the probabilistic account is currently facing.