4.2 Conditional Actions and Stochastic Policies

The interventions considered in our analysis of identification (Sections 3.3–3.4) were limited to actions that merely force a variable or a group of variables X to take on some specified value x. In general (see the process control example in Section 3.2.3), interventions may involve complex policies in which a variable X is made to respond in a specified way to some set Z of other variables—say, through a functional relationship $x = g(z)$ or through a stochastic relationship whereby X is set to x with probability $P^*(x|z)$. We will show, based on Pearl (1994b), that identifying the effect of such policies is equivalent to identifying the expression $P(y|\hat{x}, z)$.

Let $P(y|do(X = g(z)))$ stand for the distribution (of Y) prevailing under the policy $do(X = g(z))$. To compute $P(y|do(X = g(z)))$, we condition on Z and write

$$
\begin{aligned}
& P(y|do(X = g(z))) \\
&= \sum_z P(y|do(X = g(z)), z) P(z|do(X = g(z))) \\
&= \sum_z P(y|\hat{x}, z)|_{x=g(z)} P(z) \\
&= E_z[P(y|\hat{x}, z)|_{x=g(z)}].
\end{aligned}
$$

The equality

$$P(z|do(X = g(z))) = P(z)$$

stems, of course, from the fact that Z cannot be a descendant of X; hence, any control exerted on X can have no effect on the distribution of Z. Thus, we see that the causal effect of a policy $do(X = g(z))$ can be evaluated directly from the expression of $P(y|\hat{x}, z)$ simply by substituting $g(z)$ for x and taking the expectation over Z (using the observed distribution $P(z)$).

This identifiability criterion for conditional policy is somewhat stricter than that for unconditional intervention. Clearly, if a policy $do(X = g(z))$ is identifiable then the simple intervention $do(X = x)$ is identifiable as well, since we can always obtain the latter by setting

$g(z) = x$. The converse does not hold, however, because conditioning on Z might create dependencies that will prevent the successful reduction of $P(y|\hat{x}, z)$ to a hat-free expression.

A stochastic policy, which imposes a new conditional distribution $P^*(x|z)$ for x, can be handled in a similar manner. We regard the stochastic intervention as a random process in which the unconditional intervention $do(X = x)$ is enforced with probability $P^*(x|z)$. Thus, given $Z = z$, the intervention $do(X = x)$ will occur with probability $P^*(x|z)$ and will produce a causal effect given by $P(y|\hat{x}, z)$. Averaging over x and z gives

$$P(y)|_{P^*(x|z)} = \sum_x \sum_z P(y|\hat{x}, z) P^*(x|z) P(z).$$

Because $P^*(x|z)$ is specified externally, we see again that the identifiability of $P(y|\hat{x}, z)$ is a necessary and sufficient condition for the identifiability of any stochastic policy that shapes the distribution of X by the outcome of Z.

Of special importance in planning is a STRIP-like action (Fikes and Nilsson 1971) whose immediate effects $X = x$ depend on the satisfaction of some enabling precondition $C(w)$ on a set W of variables. To represent such actions, we let $Z = W \cup PA_X$ and set

$$P^*(x|z) = \begin{cases} P(x|pa_X) & \text{if } C(w) = \text{false} \\ 1 & \text{if } C(w) = \text{true and } \; X = x, \\ 0 & \text{if } C(w) = \text{true and } \; X \neq x. \end{cases}$$